MORTALITY IN THE CZECH REPUBLIC FROM 1993 TO 2010

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Abstract

This article will deal with analysis of mortality of the Czech population from the time of its independence. The aim of this analysis is to find out the changes in mortality during the reporting period. Special attention will be paid to application of models, used for extrapolating mortality curves. The Gompertz–Makeham function was the most frequented one in the last days. The new models would correspond better with the positive development of mortality. We will present the selected models and the models will be applied on obtained data about mortality in the Czech Rep. from 1993 to 2010. We will discuss about which model is the best for every particular year. At the end we will try to evaluate, if there are some changes in suitability of using models during the reporting period or not.

Key words: mortality, life expectancy, modelling of mortality

JEL Code: C10, J11

Introduction

Mortality and its development always has been very interesting topic (not only for demographers). The mortality trend is one of the most important indicators of standard of living. If people live longer, it means that mortality is going to be better. The reason for increasing life expectancy could be better health care. The second reason is greater interest in healthy life style. On the other hand the increase in values of life expectancy means population aging. Because more and more people live to the highest ages, it is very important to have the best idea of the trend of mortality at the highest ages. In the previous years it was not so important, because only a few of them live to the highest ages. It is also important to note that data about mortality at the highest ages is unreliable and mortality of oldest persons is different from the younger ones. Therefore it is necessary to use some of the existing models for extrapolating specific mortality rates at the highest ages.

1 Methodology

For extrapolating mortality there was the most frequently used Gompertz–Makeham function in the past. Later become to use modified Gompertz–Makeham function. The main reason for its application was broken assumptions about additions of the intensity of mortality at the highest ages. In these days it is clear that the Gompertz–Makeham function is better to substitute with any other existing model. Today the most preferable are logistic models.

1.1 Extrapolating of mortality curves

For extrapolating mortality curves we could use some of the existing models. We will present some of them and they will be applied on data about mortality of Czech population. The life expectancy is very frequent indicator used for describing mortality. The most important value is life expectancy at the exact age 0. Because of existing changes in the intensity of mortality during human life (e.g. Koschin [8]), it is important to look at the other values of the life expectancy (it is good to focus on critical ages). The values of life expectancy we can find in the mortality tables. Own calculation consists of several parts. The first we will calculate the specific mortality rates:

$$m_x = \frac{M_x}{\overline{S}_x}, \tag{1}$$

where M_x is the number of deaths at exact age x, $\overline{S}_x$ is the number of living. The second step is about calculation of probability of deaths and probability of survival. The other calculations are focused on hypothetical population. The last part (and the most important) is calculation of life expectancy. When we calculate specific mortality rates, it is good to know that it is important to use some of the existing models using for extrapolating mortality curves (see e.g. Fiala [8]). Logistic models are the most preferable in these days (but they are the most optimistic).

In this paper we will present Thatcher and Kannistö logistic models (e.g. Burcin et al. [2], Thatcher et al. [13], Boleslawski, Tabeau [1]). Thatcher model

$$\mu_x = \frac{z}{1+z} + \gamma, \tag{2}$$

where $z = \alpha e^{\beta x}$, α, β, γ are parameters of model and x is age.

The assumption for this model is logistic trend of mortality curve. Kannistö model has form

$$\mu_x = \frac{e^{[\theta_0 + \theta_1 (x-80)]}}{1 + e^{[\theta_0 + \theta_1 (x-80)]}}, \qquad (3)$$

where θ_0, θ_1 are parameters of model, μ_x is the intensity of mortality at exact age x. Kannistö model is special case of the logistic function, where the LOGIT transformation of mortality rates is expressed like linear function of age. The third model will be Coale–Kisker model. It is focused on the changes of mortality rates between two consecutive ages. The other assumption is that the rate of increase in mortality in the highest age decreases linearly. For these assumptions the authors created variable k_x:

$$k_x = \ln\left(\frac{m_x}{m_{x-1}}\right). \qquad (4)$$

From 85 we assume linear trend for k_x

$$k_x = k_{85} - (x - k_{85})s, \qquad (5)$$

where x is age, k_{85} and s are parameters.

For the validity of model must be complied two assumptions. Death rates around the age 85 must be as reliable for determining parameter k_{85} from the empirical data. The second assumption is associated with the mortality rate at the highest attainable age. The authors determined it up to 110 years. Known mortality rates at this age allow the estimation of parameter s. The determination of mortality rates for males and females at 110 years are based on the Swedish population mortality. The value of mortality rates were determined as 1.0 for males and 0.8 for females. The calculated model corresponds with an exponential quadratic function:

$$m_x = e^{ax^2 + bx + c}, \qquad (6)$$

where a, b and c are parameters. The last one is Gompertz–Makeham model

$$\mu_x = a + bc^x, \qquad (7)$$

where a, b and c are parameters and x is age. The Gompertz–Makeham function is used by Czech Statistical Office. The results will be compared with values of the life expectancy from life tables without extrapolation. For own calculation will be used an algorithm used for calculation of complete life tables.

2 Analysis of mortality in the Czech Republic

The aim of the presented study is to examine the changes in development of mortality in the Czech Republic from its independence. We used different disposable models which are used for extrapolating specific mortality rates. The results will be published separately for males and for females. The second part of our study includes the graphical outputs.

Tab. 1: Czech Republic – males – 1993

Model	Life expectancy at exact age – Czech Republic - males									
	0	15	20	50	65	80	85	90	95	100
Coale–Kisker	69,2	55,2	50,4	23,0	12,5	5,6	4,1	3,0	2,2	1,6
Gompertz–Makeham	69,2	55,2	50,4	23,1	12,5	5,6	4,1	3,0	2,1	1,5
Kannistö	69,3	55,2	50,4	23,1	12,5	5,7	4,3	3,2	2,5	2,0
Thatcher	69,3	55,2	50,4	23,1	12,5	5,7	4,3	3,3	2,5	2,0
Mortality table according to CZSO	69,3	55,3	50,5	23,1	12,5	5,6	4,1	3,0	2,2	1,6
Mortality table without extrapolation	69,6	56,2	51,4	23,9	13,1	5,9	4,3	3,2	2,4	1,7

Source: own calculation, (ČSÚ [3], Deras [4], Eurostat [5], Human Mortality Database [9])

Tab. 1 includes the values of the life expectancy for selected ages for males in The Czech Republic. Data are observed from 1993. At the firs point of view it is clear that the mortality tables without extrapolation give the highest values of the life expectancy. This is not correct at the highest ages. The highest values at this part of Czech males, gives for example Thatcher model (the differences begin to appear from 90 years).

Tab. 2: Czech Republic – males - 2010

Model	Life expectancy at exact age – Czech Republic - males									
	0	15	20	50	65	80	85	90	95	100
Coale–Kisker	74,3	59,6	54,8	26,6	15,2	6,6	4,5	2,9	1,8	1,0
Gompertz–Makeham	74,4	59,7	54,8	26,6	15,2	6,7	4,7	3,2	2,1	1,3
Kannistö	74,4	59,8	54,9	26,7	15,3	7,3	5,5	4,2	3,2	2,5
Thatcher	74,4	59,7	54,8	26,6	15,2	6,9	4,9	3,5	2,5	1,9
Mortality table according to CZSO	74,4	59,8	54,9	26,7	15,4	6,8	4,8	3,4	2,5	1,9
Mortality table without extrapolation	75,2	60,8	55,9	27,6	16,0	7,2	5,1	3,6	2,7	1,8

Source: own calculation, (ČSÚ [3], Deras [4], Eurostat [5], Human Mortality Database [9])

Tab. 2 includes the values of the life expectancy for males in the Czech Republic in 2010. At the first point of view it is clear that the life expectancy has increased since 1993. It means that the Czech males live longer (in average) then in the past. On the other hand it means that mortality is going to be lower. If we look carefully at the every particular model, we can see that the highest values of life expectancy we observe from Kannistö model. On the other hand the lowest values give Coale–Kisker model.

Tab. 3: Czech Republic – females - 1993

Model	Life expectancy at exact age – Czech Republic - females									
	0	15	20	50	65	80	85	90	95	100
Coale–Kisker	76,4	62,2	57,3	28,6	15,9	6,6	4,5	3,0	1,9	1,2
Gompertz–Makeham	76,4	62,2	57,3	28,6	15,9	6,6	4,5	3,0	1,9	1,2
Kannistö	76,5	62,3	57,4	28,7	16,0	6,8	4,8	3,4	2,5	1,9
Thatcher	76,5	62,2	57,3	28,6	16,0	6,7	4,8	3,4	2,4	1,8
Mortality table according to CZSO	76,5	62,3	57,3	28,6	16,0	6,6	4,7	3,3	2,3	1,6
Mortality table without extrapolation	76,9	63,2	58,3	29,5	16,7	7,1	5,0	3,5	2,5	1,8

Source: own calculation, (ČSÚ [3], Deras [4], Eurostat [5], Human Mortality Database [9])

From the Tab. 3 we can see the similar situation like for Czech males. The highest values of life expectancy are in the mortality tables without extrapolation. This is not correct for every single age like for Czech males. The values of the Kanistö model begin to exceed the values of life tables without extrapolation from 100 years of life. It is very interesting that the situation is different from Czech males, because the situation here changes approximately ten years later.

Tab. 4: Czech Republic – females - 2010

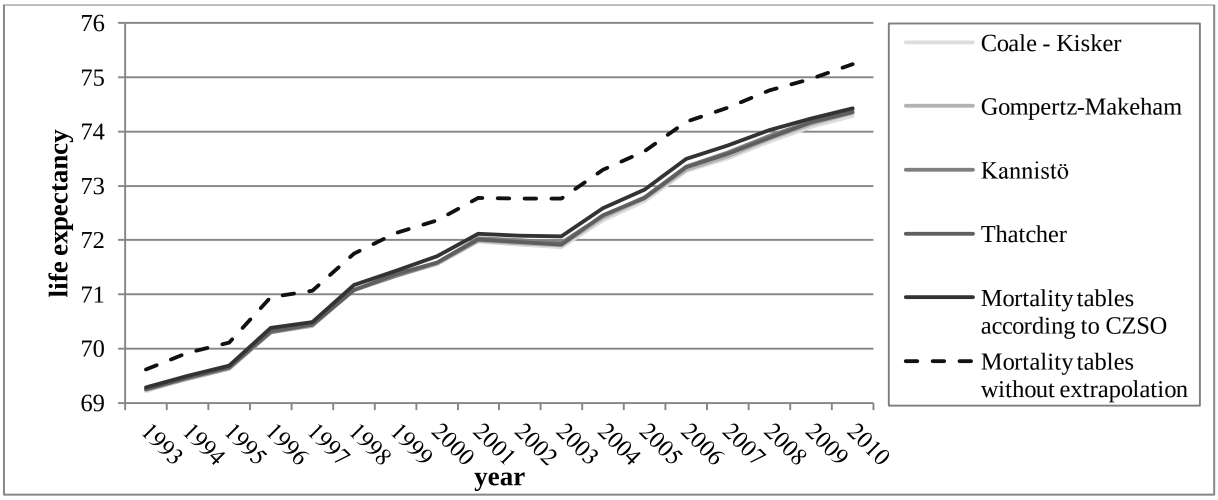
Model	Life expectancy at exact age – Czech Republic - females									
	0	15	20	50	65	80	85	90	95	100
Coale–Kisker	80,4	65,8	60,8	31,7	18,5	7,7	5,0	3,0	1,6	0,8
Gompertz–Makeham	80,5	65,9	60,9	31,8	18,7	7,9	5,3	3,3	2,0	1,1
Kannistö	80,8	66,1	61,1	32,0	18,9	8,5	6,1	4,3	3,1	2,2
Thatcher	80,7	66,0	61,1	31,9	18,8	8,0	5,5	3,7	2,5	1,8
Mortality table according to CZSO	80,3	65,7	60,7	31,7	18,6	7,9	5,4	3,8	2,6	1,9
Mortality table without extrapolation	81,1	66,7	61,7	32,6	19,4	8,4	5,8	4,0	2,8	1,9

Source: own calculation, (ČSÚ [3], Deras [4], Eurostat [5], Human Mortality Database [9])

Tab. 4 includes the life expectancy for Czech females in 2010. We can see the life expectancy has increased. So it means that the mortality is going to be lower.

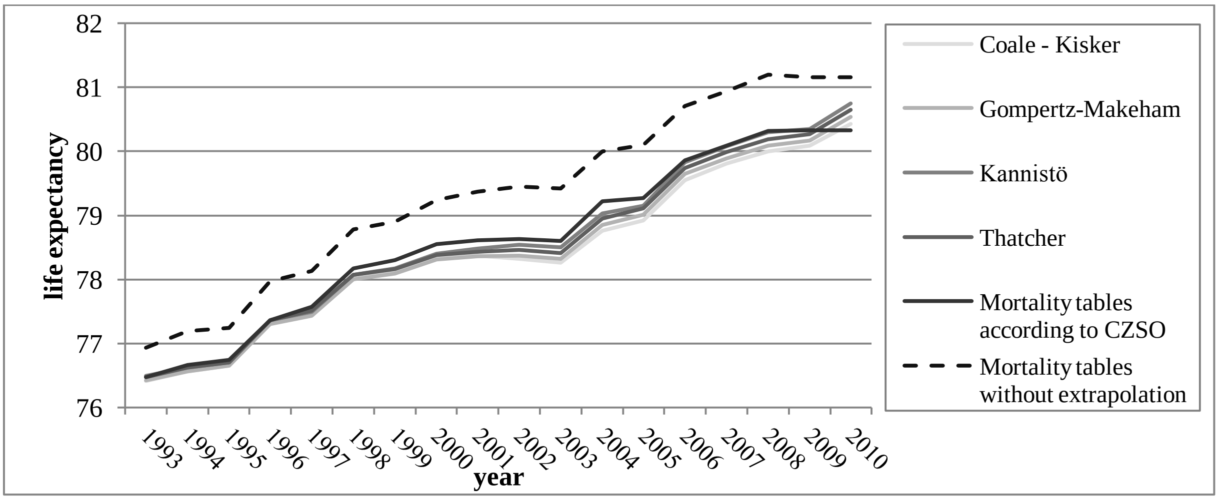
The first graph in Fig. 1 shows us the development of life expectancy at exact age 0 for Czech males. It is clear that the life expectancy has increased (this is correct for every single model). The highest values give the mortality tables without extrapolation. If we look more carefully at obtain results, we can see that the most optimistic is Kannistö or Thatcher model (they gives the highest values of life expectancy). On the other hand, the pessimistic models are Gompertz–Makeham or Coale–Kisker model.

Fig. 1: Life expectancy for males in the Czech Rep. at exact age 0 from 1993 to 2010



Source: own construction

Fig. 2: Life expectancy for females in the Czech Rep. at exact age 0 from 1993 to 2010



Source: own construction

If we compare the results for males and for females in the Czech Republic, we find out that the life expectancy at exact age 0 for both sexes are increasing. It means that the mortality is going to be better in the Czech Republic in general. But it is very important to know, that the increase of life expectancy means the aging of the population. If we look more carefully at the every particular model, we can make the similar conclusion. On the other hand, between the most pessimistic models we can include the Gomperz–Makeham or Coale–Kisker model. The most optimistic is for example Kannistö or Thatcher model.

Conclusion

From our results it is clear that the life expectancy at exact age *x* has increased both for males and females. We get higher values of life expectancy for Czech females (this is correct for the whole reporting period). Our results correspond with original assumption (females live longer in average then males). The increasing trend of life expectancy means the improvement in mortality for both sexes. But it has also the negative side – the aging of the population. If we look more carefully at the results obtained for every single model and if we compare them with morality tables without extrapolation, we find out, that the Kannistö (or Thatcher) model gives the highest values of life expectancy. These models are the most optimistic. On the other hand, between the most pessimistic (according to obtained values of life expectancy) belong Gompertz–Makeham or Coale–Kisker model. If we are interested in which model is the best for describing mortality rates, we can say, that the best one could be Kannistö (or Thatcher) model. But if we make a decision, which model is good to use, we have to know that the suitability of using model is dependent on mortality of analysed population.

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