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DETERMINISTIC AND STOCHASTIC EXTRAPOLATION OF DEATH RATES IN THE CZECH REPUBLIC WITH AN IMPACT ON PROBABILITY OF DYING AND TABULAR NUMBER OF DEATHS

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Abstract. The aim of this paper is to compare the results of deterministic (linear regression) and stochastic (autoregressive integrated moving averages) extrapolations of logarithms of age-and-sex-specific death rates of the Czech population with impact on key characteristics of mortality tables – probability of dying and tabular number of deaths. The result is that the optimized stochastic ARIMA models provide more realistic results about the future development of the analysed characteristics and they are much less biased. Deterministic models have problems in applications on variable data, which can be expected especially in the highest ages.

Keywords: deterministic models, stochastic models, mortality, death rates, projection

Mathematics Subject Classification: Primary 90C30; Secondary 62H12

1 Introduction

Information about the future development of the age-and-sex specific structure of the population is very important for the central and regional administration. Prospective trend estimates of the number and structure of the population have their demographic importance, especially shows the prospective population consequences of the current population development. In practice, the population projections are either based on common mathematical extrapolation methods or methods of demographic population projections (Booth et al. [5]). The individual results depend on the procedure for calculating of demographic projection. Population structure and all derived characteristics are strongly determined by applied scenario and also by the model, which was used for extrapolation the future age-and-sex-specific demographic characteristics (Erbas et al. [11]). Future development for individual demographic processes can have an optimistic or pessimistic nature, resulting to the population projection in optimistic or pessimistic variant.

The aim of this paper is to focus on deterministic (which was used in the past and still uses) and stochastic (alternative) extrapolating of death rates, and compare the results between these approaches. The database is available for the Czech Republic at the age of 0–100+ completed years of life and the period from 1920 to 2012. We pay the attention to the differences that provides deterministic and stochastic model and compare the impacts of their applications.

2 Materials and Methods

For the description of mortality development is most often used an indicator known as life expectancy, (as a key output of the mortality tables), but for analytical purposes may be used other characteristics of mortality, such as e.g. age-and-sex specific death rates, probability of dying and the tabular number of deaths. Nowadays, the development of mortality for the oldest persons (and thus the development of life expectancy and other characteristics resulting from mortality tables) enters to the forefront of research interest (see e.g. paper by Bell [3]). The extension of human life is still in progress. It is important to note that for the examining of mortality at the highest ages it is difficult to use an empirical data, because of significant variability. It is caused by the different development of mortality trend in the highest age groups compared to younger ones. Probability of dying and tabular number of deaths is obtained as an output indicator from mortality tables. The calculation is carried out in several steps. First, we calculate the age-and-sex specific death rates as

$$m_{x,t} = \frac{M_{x,t}}{\overline{S}_{x,t}}, \tag{1}$$

where $M_{x,t}$ is the number of deaths at the exact age x and $\overline{S}_{x,t}$ is the mid-year number of living. Between the specific mortality rate and the intensity of mortality (which is considered in similar studies and analysis, see e.g. paper by Bogue, Anderton, Arriaga [4], Fiala [12] or Dotlačilová, Šimpach [9], [10]) is valid the followed relationship

$$m_{x,t} \approx \mu\left(x + \frac{1}{2}\right). \tag{2}$$

Next, we will calculate the probability of death for 0-year-old person as

$$q_{0,t} = \frac{M_{0,t}}{\alpha N_t^v + (1-\alpha) N_{t-1}^v} \quad \text{for } x = 0, \tag{3}$$

where $M_{0,t}$ is the number of deaths at the age 0, α is the proportion of lower elementary file of died persons (approximately 0.85) and N_t^v, respectively N_{t-1}^v is the number of live births in year t, respectively in year t-1 (Keyfitz [14]). The probability of dying

$$q_{x,t} = 1 - p_{x,t}, \tag{4}$$

is valid for $x > 0$. The calculation of the probability of surviving is given by formula

$$p_{0,t} = 1 - q_{0,t} \quad \text{for } x = 0, \tag{5}$$

and by formula

$$p_{x,t} = e^{-m_{x,t}} \text{ for } x > 0. \tag{6}$$

Next part of the calculation relates to tabular (i.e. imaginary) population. First, we select the initial number of live births in tabular population: $l_{0,t}$ = 100 000. Based on knowledge of the probability of surviving, we are able to calculate the number of survivors in the further exact ages by

$$l_{x+1,t} = l_{x,t} p_{x,t} , \tag{7}$$

where $l_{x,t}$ is the number of survivors at the exact age x from the default file of 100 000 live births of tabular population. The number of deaths of tabular population is given by

$$d_{x,t} = l_{x,t} q_{x,t} . \tag{8}$$

In the following steps it possible to calculate the number of lived years ($L_{x,t}$), the number of remaining years of life ($T_{x,t}$) and life expectancy ($e_{x,t}$), but because these characteristics are not used in the paper, they will not be explained here.

Extrapolation approaches applied on the mortality development, which are based on a deterministic principles are still used in the conditions of the Czech Republic. We can mention e.g. study by Fiala [12] or Koschin et al. [15]. Especially the study by Fiala [12] provides information that $\ln(m_{x,t})$ is for each age x approximately linear and can be modelled by linear or polynomial function (according to significance of parameters). According to this evidence we can conclude that if we find the appropriate constant (intercept) (β_0) and slope (β_1) of the regression line, we can easily perform linear extrapolation to the future as

$$\ln(m_{x,t}) = \beta_0 + \beta_1 t + \varepsilon_t , \tag{9}$$

where t is the calendar year and predict the future development of mortality rates. This approach is simple, however, because the database of the Czech Republic is quite variable, and the nature of individual time series of mortality is volatile, residuals of these regressions can be autocorrelated and the slope of the regression line (especially in the lowest and highest age groups, where is present the highest variability in the data) may not be optimal for achieving the desired results.

Another approach which was used e.g. by Arltová et al. [2], Šimpach [19], [20] or Šimpach, Pechrová [21] is based on Box and Jenkins [6] methodology, where the linear regression model (9) is replaced by autoregressive integrated moving averages model ARIMA, which can be written as

$$\ln m_{x,t} = \beta_0 + \sum_{i=1}^{p} \phi_i \ln m_{x,t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} , \tag{10}$$

where the autoregressive part is

$$\ln m_{x,t} = \beta_0 + \sum_{i=1}^{p} \phi_i \ln m_{x,t-i} + \varepsilon_t . \tag{11}$$

This is the model of order p AR(p), where ϕ_i (i = 1, ..., p) are the parameters, β_0 represents the constant, and ε_t is a pure white noise error term, where $E(\varepsilon_t) = 0$, $D(\varepsilon_t) = \sigma^2$, $cov(\varepsilon_t ; \varepsilon_t') = 0$ and $\varepsilon_t \approx$ N (normal) distribution (Mélard, Pasteels [16] or Ord, Lowe [17]. Moving average model of order q is formulated as

$$\ln m_{x,t} = \varepsilon_1 + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} \,, \tag{12}$$

where θ_j (j = 1, ..., q) are the parameters and ε_t is a white noise.

Followed by the above mentioned procedure it is obvious, that in the first step we estimate 101 linear regression models for males (age 0–100+ years of life) and 101 models for females with use of database of logarithms of age-specific death rates (see also Stauffer [18]). Then, these models are used for extrapolation up to the year 2100, (in the same way as it performs e.g. CZSO [8]). In the second step there are calculated new 2 × 101 models using an automatized process for time series modelling in Statgraphics Centurion XVI. ARIMA models with optimally selected parameters (Akaike [1]) are used to extrapolate the logarithms of age-specific death rates, also up to the year 2100. In the third step, there are the empirical and predicted death rates recalculated to probabilities of dying and table number of deaths using an algorithm of mortality tables.

3 Results

Estimated intercepts (β_0) and slopes (β_1) of the regression lines for logarithms of age-specific death rates of male and female population in the Czech Republic are presented in Tab. 3 in Appendix. Extrapolation is connected to empirical data, and the results are presented in Fig. 1 (left) for males and in Fig. 2 (left) for females. Optimized forms (see also [16] or [17]) of ARIMA models for these logarithms are shown in Tab. 1 below (for males) and in Tab. 2 below (for females).

Age	Model	Age	Model	Age	Model	Age	Model	Age	Model	Age	Model	Age	Model	Age	Model
0	(0,2,1) c	13	(0,1,1) c	26	(0,1,1) c	39	(0,1,1) c	52	(0,1,1) c	65	(0,1,1) c	78	(0,1,1) c	91	(0,1,2) c
1	(0,1,1) c	14	(0,1,1) c	27	(0,1,1) c	40	(2,2,1)	53	(0,1,1) c	66	(0,1,1) c	79	(1,1,1) c	92	(2,1,0) c
2	(2,2,1)	15	(0,1,1) c	28	(0,1,1) c	41	(0,1,1) c	54	(0,1,1) c	67	(0,1,1) c	80	(0,1,1) c	93	(0,1,1) c
3	(0,1,1) c	16	(0,1,1) c	29	(0,1,1) c	42	(0,1,1) c	55	(0,1,1) c	68	(0,1,1) c	81	(0,1,1) c	94	(0,1,2) c
4	(0,1,1) c	17	(0,1,1) c	30	(0,1,1) c	43	(0,1,1) c	56	(2,1,0) c	69	(0,1,1) c	82	(0,1,1) c	95	(1,1,1) c
5	(0,1,1) c	18	(0,1,1) c	31	(2,2,1)	44	(0,1,1) c	57	(0,1,1) c	70	(0,1,1) c	83	(1,1,1) c	96	(0,1,2) c
6	(0,1,1) c	19	(0,1,1) c	32	(0,1,1) c	45	(0,1,1) c	58	(0,1,1) c	71	(0,1,1) c	84	(0,1,1) c	97	(0,1,1) c
7	(0,1,1) c	20	(0,1,2) c	33	(0,1,1) c	46	(0,1,1) c	59	(0,1,1) c	72	(0,1,1) c	85	(0,1,1) c	98	(0,1,1) c
8	(0,1,1) c	21	(0,1,1) c	34	(0,1,1) c	47	(2,1,0) c	60	(0,1,1) c	73	(1,1,0) c	86	(0,1,1) c	99	(0,1,2) c
9	(0,1,1) c	22	(0,1,1) c	35	(0,1,1) c	48	(1,1,0) c	61	(0,1,1) c	74	(1,1,0) c	87	(0,1,1) c	100+	(0,1,1) c
10	(0,1,1) c	23	(0,1,1) c	36	(0,1,1) c	49	(0,1,1) c	62	(0,1,1) c	75	(2,1,0) c	88	(0,1,1) c		
11	(0,1,1) c	24	(0,1,1) c	37	(0,1,1) c	50	(2,1,0) c	63	(0,1,1) c	76	(0,1,1) c	89	(0,1,2) c		
12	(0,1,1) c	25	(2,1,0) c	38	(0,1,1) c	51	(1,1,0) c	64	(0,1,1) c	77	(0,1,1) c	90	(2,1,0) c		

Tab. 1. ARIMA (p, d, q) models with or without constant for logarithms of male’s age-specific death rates in range of 0–100+ years in the Czech Republic. Source: authors’ calculations and construction.

Age	Model	Age	Model	Age	Model	Age	Model	Age	Model	Age	Model	Age	Model	Age	Model
0	(0,2,1) c	13	(0,1,1) c	26	(0,1,1) c	39	(0,1,1) c	52	(0,1,1) c	65	(1,1,0) c	78	(2,1,2) c	91	(1,1,1) c
1	(0,1,1) c	14	(1,1,1) c	27	(0,1,1) c	40	(2,1,0) c	53	(0,1,1) c	66	(1,1,0) c	79	(0,1,1) c	92	(2,1,0) c
2	(1,1,1) c	15	(0,1,1) c	28	(0,1,1) c	41	(1,1,0) c	54	(0,1,1) c	67	(0,1,1) c	80	(0,1,1) c	93	(0,1,1) c
3	(2,1,0) c	16	(2,1,0) c	29	(0,1,1) c	42	(0,1,1) c	55	(0,1,1) c	68	(0,1,1) c	81	(0,1,1) c	94	(2,1,1) c
4	(2,1,0) c	17	(0,1,1) c	30	(0,1,1) c	43	(0,1,1) c	56	(0,1,1) c	69	(0,1,1) c	82	(0,1,1) c	95	(1,0,0) c
5	(0,1,1) c	18	(0,1,2) c	31	(0,1,1) c	44	(0,1,1) c	57	(0,1,1) c	70	(0,1,1) c	83	(0,1,1) c	96	(1,0,0) c
6	(1,1,1) c	19	(0,1,1) c	32	(0,1,1) c	45	(0,1,1) c	58	(2,1,0) c	71	(1,1,0) c	84	(0,1,1) c	97	(0,0,2) c
7	(0,1,1) c	20	(0,1,1) c	33	(0,1,1) c	46	(0,1,1) c	59	(1,1,1) c	72	(2,1,0) c	85	(0,1,1) c	98	(0,0,2) c
8	(2,1,0) c	21	(0,1,1) c	34	(0,1,1) c	47	(0,1,1) c	60	(0,1,1) c	73	(1,1,0) c	86	(0,1,1) c	99	(1,1,0) c
9	(2,1,0) c	22	(2,1,0) c	35	(2,1,0) c	48	(0,1,1) c	61	(0,1,2) c	74	(2,1,0) c	87	(0,1,1) c	100	(0,1,1) c
10	(0,1,1) c	23	(0,1,1) c	36	(0,1,1) c	49	(1,1,1) c	62	(1,1,0) c	75	(2,1,0) c	88	(0,1,1) c		
11	(0,1,1) c	24	(1,1,0) c	37	(0,1,1) c	50	(0,1,1) c	63	(1,1,0) c	76	(0,1,1) c	89	(1,1,1) c		
12	(1,1,1) c	25	(0,1,1) c	38	(0,1,1) c	51	(0,1,1) c	64	(0,1,1) c	77	(0,1,1) c	90	(2,1,0) c		

Tab. 2. ARIMA (p, d, q) models with or without constant for logarithms of female's age-specific death rates in range of 0–100+ years in the Czech Republic. Source: authors' calculations and construction.

Forecasts of death rates by ARIMA models are connected to empirical data, and the results are presented in Fig. 1 (right) for males and in Fig. 2 (right) for females. It is clear that the lower values of logarithms of these rates provide ARIMA models (in comparison with deterministic approach of linear regression). According to diagnostic tests (Jarque, Bera [13]) and comparison with other results (see e.g. Šimpach [20]) and theoretical expectations, these parameters are not deviated and probably provide more optimistic results.

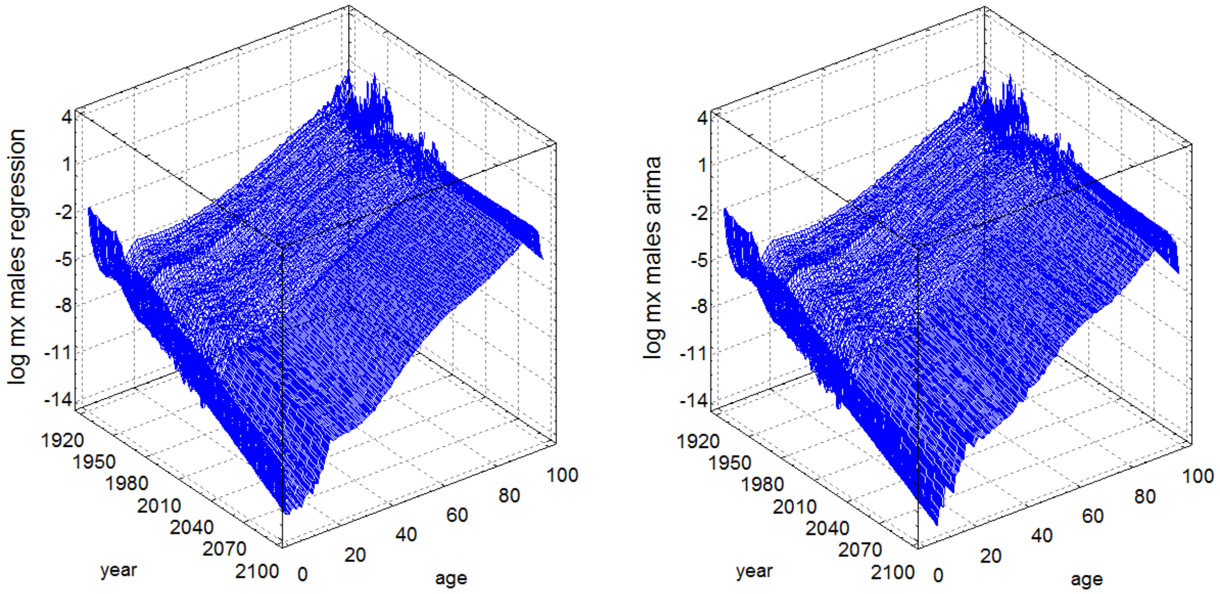

Fig. 1. Empirical values of age-specific death rates in logarithms of Czech males from 1920 to 2012 with attached extrapolated values up to the year 2100 according to individual linear regression models (left) and ARIMA models (right). Source: authors' calculations and construction

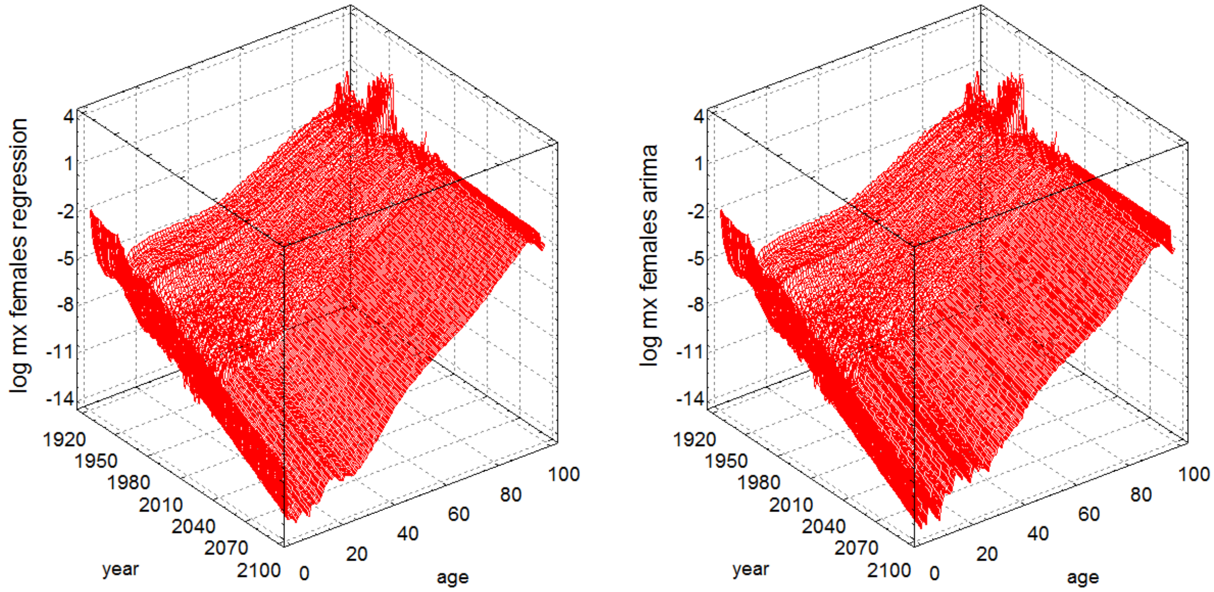

Fig. 2. Empirical values of age-specific death rates in logarithms of Czech females from 1920 to 2012 with attached extrapolated values up to the year 2100 according to individual linear regression models (left) and ARIMA models (right). Source: authors' calculations and construction

Larger differences between the results of linear regression and stochastic ARIMA models can be seen after recalculation of age-specific death rates to the probability of dying (Fig. 3 and 4). Due to the extreme values in empirical data of logarithms of death rates there is a bias in the highest ages and unreasonable escalation of probability of dying in the age of 90+. ARIMA models were not biased and provide more realistic results (see mainly the differences in highest age groups). In the case of linear models and population of males is the situation worse – the estimates are more biased.

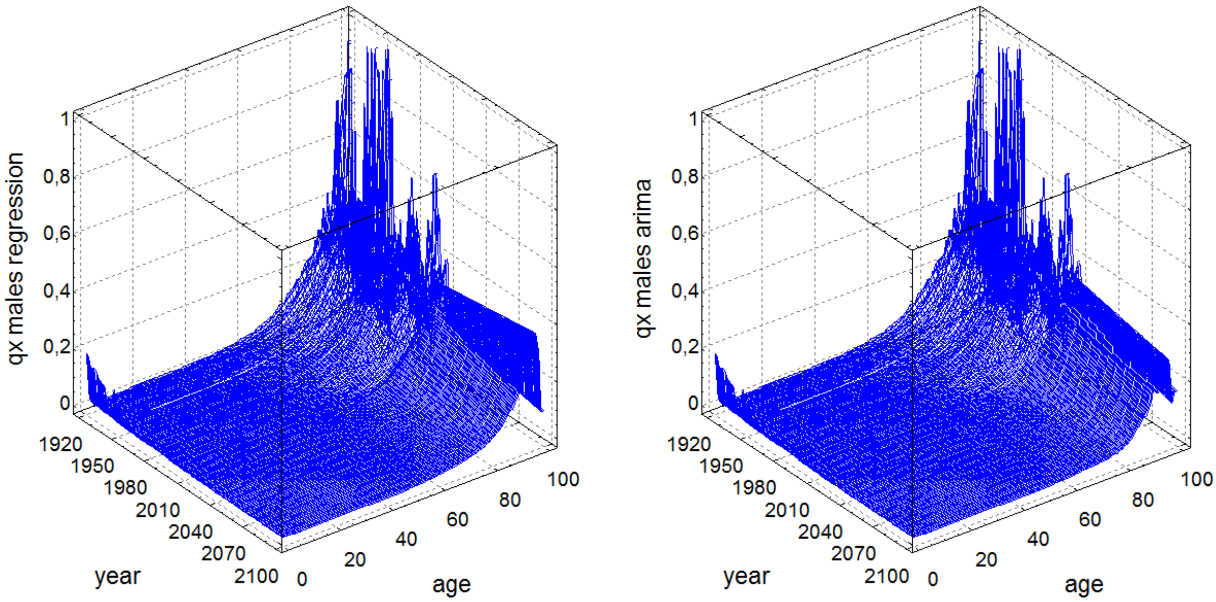

Fig. 3. Empirical values of age-specific probabilities of dying of Czech males from 1920 to 2012 with attached extrapolated values up to the year 2100 according to linear regression models (left) and ARIMA models (right) applied on log. of age-specific death rates. Source: authors' calculations and construction

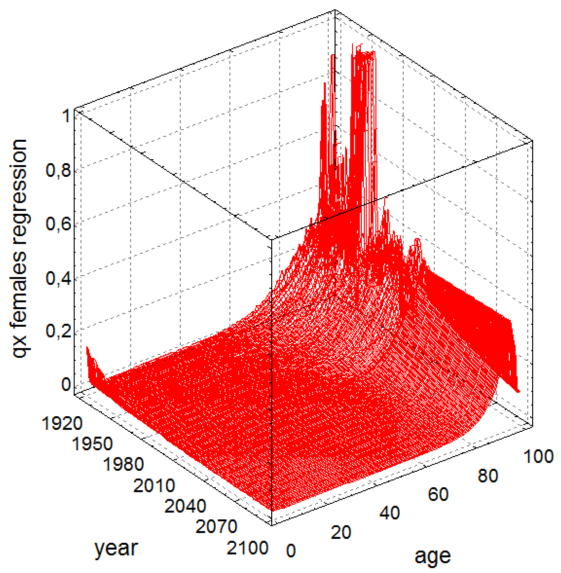

Fig. 4. Empirical values of age-specific probabilities of dying of Czech females from 1920 to 2012 with attached extrapolated values up to the year 2100 according to linear regression models (left) and ARIMA models (right) applied on log. of age-specific death rates. Source: authors' calculations and construction

In the last part of analysis we recalculated and extrapolated the empirical data of death rates to the table number of deaths on the basis of mortality tables algorithm. Not only that stochastic models predict moving of the mode of the deceased persons to the higher ages, but also sharper distribution of these numbers – i.e. that the concentration around the mode is denser. Much more important is this conclusion in the case of female population. While the tabular number of deaths in the case of the male population starts significantly growing from 62 years, for female population starts this increasing from 78 years. In all cases, the predicted values are continuously connected to the empirical data.

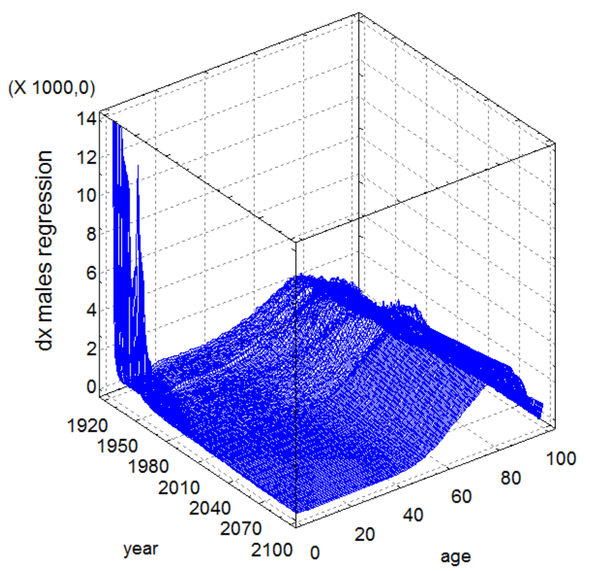

Fig. 5. Empirical values of age-specific tabular number of deaths of Czech males from 1920 to 2012 with attached extrapolated values up to the year 2100 according to linear regression models (left) and ARIMA models (right) applied on log. of age-specific death rates. Source: authors' calculations and construction

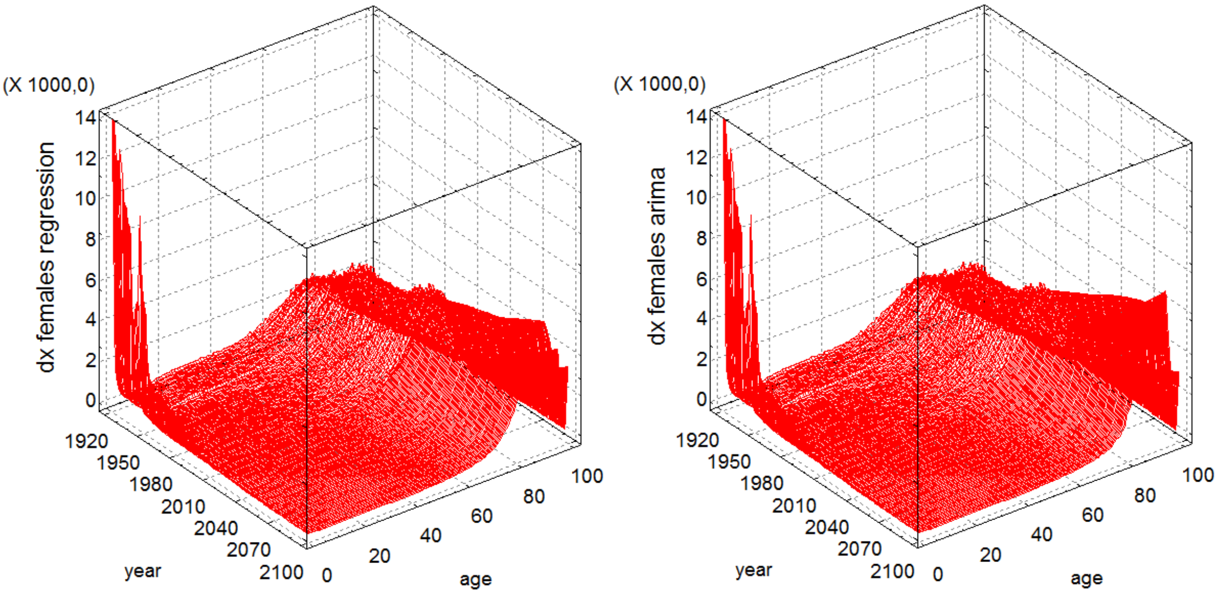

Fig. 6. Empirical values of age-specific tabular number of deaths of Czech females from 1920 to 2012 with attached extrapolated values up to the year 2100 according to linear regression models (left) and ARIMA models (right) applied on log. of age-specific death rates. Source: authors' calculations and construction

4 Discussion and Conclusion

Demographic projections of possible future population development are the basic information that is used to provide key information about the potential development of mortality (or fertility, migration and other demographic processes). Each projection is based on assumptions, which to a certain extent may or may not occur. The aim of this paper was to compare the results of deterministic extrapolation of logarithms of age-specific death rates of the Czech population with the results of extrapolation using stochastic ARIMA models of Box-Jenkins [6] methodology. We used data from the CZSO [7] database in the age range of 0–100+ completed years of life for each sex and because we used two types of methodological approach, there was designed 2 × 2 × 101 = 404 individual models. On the basis of these models we made an extrapolation of logarithms of age-specific death rates up to the year 2100. Using by mortality tables algorithm the death rates were recalculated to the probability of dying and table numbers of deaths.

Results showed that extreme values, which contain analysed time series of logarithms of death rates, (mainly in the case of lowest and highest age groups), caused deflection of the estimated parameters of linear regression models, and thus the deflection of extrapolation of these rates to the future. Results are situated in the range in which it should not be placed. Stochastic models are much more robust for these extreme values. The extrapolation is not deflected, and the results are more relevant. Therefore it is obvious that the individual time series of logarithms of age-specific death rates contain a stochastic trend, which is explainable using by ARIMA models with optimized structure of parameters.

From the forecasted results of probabilities of dying and table number of deaths it is clear, that the linear regression models and stochastic ARIMA models are usable for age range approximately of

2–60 years. For ages 0, 1, and 61+ can be used stochastic ARIMA models only, because these models are robust in the case of strong variability and extreme values.

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Appendix

Tab. 3 shows the estimates of intercepts and slopes of 2 × 101 individual linear regression models.

MALES						FEMALES					
Age	Intercept	Slope	Age	Intercept	Slope	Age	Intercept	Slope	Age	Intercept	Slope
0	88,5109	-0,0468	51	9,6868	-0,0073	0	89,4360	-0,0474	51	24,9175	-0,0153
1	93,1204	-0,0505	52	9,0396	-0,0069	1	95,6204	-0,0518	52	24,6025	-0,0151
2	83,7540	-0,0460	53	8,5326	-0,0066	2	86,0081	-0,0473	53	23,5246	-0,0145
3	77,0660	-0,0427	54	7,0759	-0,0058	3	83,1380	-0,0459	54	22,8473	-0,0141
4	72,7697	-0,0406	55	6,9301	-0,0057	4	83,7996	-0,0464	55	22,2736	-0,0138
5	73,7969	-0,0412	56	6,7031	-0,0055	5	78,5892	-0,0438	56	22,1037	-0,0137
6	69,9993	-0,0393	57	6,0191	-0,0051	6	73,8460	-0,0414	57	21,1151	-0,0131
7	71,9836	-0,0404	58	5,6661	-0,0049	7	70,4510	-0,0397	58	21,1456	-0,0131
8	63,8499	-0,0363	59	5,5056	-0,0048	8	72,4563	-0,0408	59	20,5336	-0,0127
9	59,8880	-0,0343	60	5,4730	-0,0047	9	74,7851	-0,0421	60	21,2484	-0,0130
10	55,9073	-0,0323	61	4,6096	-0,0042	10	66,0924	-0,0376	61	20,7178	-0,0127
11	59,6766	-0,0342	62	4,4959	-0,0041	11	64,8665	-0,0370	62	21,6291	-0,0131
12	52,2472	-0,0304	63	4,4290	-0,0040	12	66,2089	-0,0377	63	21,0899	-0,0128
13	53,2289	-0,0309	64	4,7992	-0,0042	13	65,2777	-0,0372	64	21,4866	-0,0130
14	48,1625	-0,0282	65	5,0595	-0,0043	14	62,9035	-0,0359	65	20,9945	-0,0127
15	47,7204	-0,0279	66	5,0000	-0,0042	15	58,3992	-0,0335	66	21,0429	-0,0126
16	43,2882	-0,0255	67	5,0075	-0,0042	16	58,9243	-0,0337	67	21,1877	-0,0126
17	40,0813	-0,0238	68	5,2287	-0,0042	17	57,1798	-0,0328	68	20,9037	-0,0124
18	33,7400	-0,0204	69	5,3565	-0,0042	18	60,7097	-0,0345	69	20,7745	-0,0123
19	37,1909	-0,0222	70	5,9055	-0,0045	19	58,7712	-0,0335	70	20,9959	-0,0124
20	36,9525	-0,0220	71	5,7020	-0,0043	20	61,6050	-0,0349	71	20,8525	-0,0123
21	36,7412	-0,0219	72	6,4518	-0,0047	21	65,4689	-0,0369	72	20,7576	-0,0121
22	36,6589	-0,0218	73	6,8503	-0,0048	22	66,3813	-0,0374	73	20,2750	-0,0118
23	35,9527	-0,0215	74	6,9074	-0,0048	23	63,9931	-0,0361	74	19,8773	-0,0116
24	35,3041	-0,0211	75	6,8421	-0,0047	24	64,5962	-0,0364	75	18,9277	-0,0110
25	36,0789	-0,0216	76	6,9110	-0,0047	25	65,5761	-0,0369	76	18,5266	-0,0108
26	34,4657	-0,0207	77	7,1497	-0,0048	26	66,9680	-0,0376	77	18,3157	-0,0106
27	34,2254	-0,0206	78	7,1250	-0,0047	27	66,2216	-0,0372	78	16,9967	-0,0099
28	34,1289	-0,0206	79	7,3828	-0,0048	28	63,2096	-0,0357	79	16,8866	-0,0098
29	33,0804	-0,0200	80	7,3361	-0,0048	29	63,2161	-0,0357	80	16,0546	-0,0093
30	33,0810	-0,0200	81	7,2687	-0,0047	30	60,4873	-0,0342	81	14,8764	-0,0087
31	30,9401	-0,0189	82	6,8742	-0,0044	31	58,8892	-0,0334	82	14,2864	-0,0083
32	30,7952	-0,0188	83	7,4319	-0,0047	32	55,8598	-0,0318	83	13,6961	-0,0079
33	29,9794	-0,0183	84	7,3120	-0,0046	33	54,7079	-0,0312	84	13,0070	-0,0075
34	29,9375	-0,0183	85	6,4634	-0,0041	34	51,4006	-0,0295	85	12,4514	-0,0072
35	29,4114	-0,0180	86	6,1661	-0,0039	35	49,8194	-0,0287	86	11,9430	-0,0069
36	26,7796	-0,0166	87	5,5515	-0,0035	36	48,0991	-0,0277	87	10,2922	-0,0060
37	25,5494	-0,0160	88	5,9894	-0,0037	37	46,0115	-0,0266	88	9,5985	-0,0056
38	23,3905	-0,0148	89	5,1864	-0,0033	38	44,0519	-0,0256	89	8,8090	-0,0052
39	22,7481	-0,0145	90	4,2055	-0,0028	39	41,6228	-0,0243	90	7,2629	-0,0044
40	22,3299	-0,0142	91	2,9028	-0,0021	40	39,8129	-0,0234	91	5,2910	-0,0033
41	20,8742	-0,0134	92	1,8045	-0,0015	41	37,1529	-0,0220	92	5,0704	-0,0032
42	19,6673	-0,0128	93	1,1032	-0,0011	42	34,3563	-0,0205	93	4,5636	-0,0029
43	17,8824	-0,0118	94	1,7199	-0,0014	43	32,6876	-0,0196	94	5,4427	-0,0033
44	17,0943	-0,0114	95	3,8211	-0,0024	44	31,1862	-0,0188	95	11,8602	-0,0064
45	16,6479	-0,0111	96	1,5766	-0,0013	45	30,9918	-0,0187	96	-2,0406	0,0006
46	14,1438	-0,0098	97	-4,9009	0,0020	46	28,3308	-0,0173	97	1,7633	-0,0013
47	13,2588	-0,0093	98	-1,6089	0,0003	47	27,3924	-0,0168	98	6,1229	-0,0035
48	12,5806	-0,0089	99	15,4148	-0,0084	48	26,6438	-0,0163	99	20,6265	-0,0109
49	11,0328	-0,0080	100	19,2148	-0,0105	49	26,3721	-0,0162	100	9,0942	-0,0053
50	10,6577	-0,0078				50	25,7097	-0,0158			

Tab. 3. Estimated parameters (intercepts and slopes) of linear regression models for logarithms of males' and females' age-specific death rates in range of 0–100+ years in the Czech Republic.
Source: authors' calculations and construction.

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